Gaudí-inspired Generative Quilts
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Abstract

In *Generative Art 2015*, we presented quilted textile works incorporating generative tilings printed directly to a fabric substrate; the resulting 2.5D surface textures of the quilted fabrics inspired our current work. In a departure from figurative work printed to fabric, we now present quilted generative abstractions in layered fabrics and thread, using batik and organza. These include catenary arches from a 3D space, collapsed onto the 2.5D space provided by a traditional quilt. The individual stitches of a piecewise-linear quilted rendering of an analytical curve echo the individual chain links which inspired the analytical catenary solution long ago.

Wolfram Research's *Mathematica* language is used to create generative line drawings with an aesthetic-heuristic selection criteria. These are rendered with corresponding surfaces of revolution in 3D, and these ruled surfaces are projected onto a planar representation. The corresponding SVG (Scalable Vector Graphic) format exchange file is realized on an ABM Innova (http://www.abminternational.com/) quilting machine using *Art and Stitch* (http://artandstitch.com/) software to render the images in fabric and thread.

This body of work was inspired by Antoni Gaudí's string, rope and lead weight models used to design the arches of La Sagrada Família in Barcelona. A recent rendition of Gaudí's catenaries was created in 2012 by Sarah van Gameren and Tim Simpson (a.k.a. Glithero) in *Lost Time*, a construction of ball bearings on string hanging over a reflecting pool.

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Main References:

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Gaudí-inspired Generative Quilts

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Form and Inspiration

For *Generative Art 2015*, we presented one in a series of quilts in *Juxtaposes: Visual Granular Synthesis, Vernacular Architecture and Girih Tilings*. In that body of work, quilting stiches followed the outlines of patterns and an urban landscape montage printed to fabric. In this work, we focus entirely on the surface created by the stitching, with no underlying image to follow. The fabric is secondary to the process; surface design is paramount.

The surface pattern is inspired by Antoni Gaudí's chain or string and lead weight models of catenary arches used in the design of La Sagrada Família and Casa Mila in Barcelona. "A catenary arch is the shape one gets when we suspend a rope or chain from its endpoints. …The advantage of the catenary arch is that it can be constructed from relatively light materials while still being able to support great weights."[1]

In Gaudi's chain model displayed at Casa Mila, "if a chain end-point is moved" to change the floor plan "in one corner, then the shape of the entire hanging chain model shifts and settles into a newly optimized catenary geometry."[i] Similarly, because the quilted line is one continuous line, if one endpoint of the design is shifted, the entire form changes. Another affinity between a chain assembly and quilted construction is that a stitched curve is also piecewise linear; stitches in a quilt echo the individual chain links forming a catenary curve. An additional inspiration for the quilts was a recent rendition of Gaudí's catenaries created in 2012 by Sarah van Gameren and Tim Simpson (a.k.a. Glithero) in *Lost Time*,[ii] a construction of ball bearings on string hanging over a reflecting pool.

In this work, ruled line drawings of catenary arches from a 3D space are collapsed onto the 2.5D space provided by a traditional quilt. Once rendered in stitching, the spatial ambiguities that Celestino Soddu called "contaminations between different dimensions"[iii] in the line drawing are compounded by additional surface effects. Closely spaced sewn lines appear flatter, and hence more distant. Conversely, lines with greater spacing create a *trapunto* effect (*i.e.*, raised relief from the ground), perceptually pulling these elements to the foreground. Contaminations occur when there are catenaries of different scales or near forms overlap several flattened far forms. Catenaries of different sizes can either signal a spatial separation in depth or they can simply be larger and smaller forms.

Generative Process

Background

Our quilts are prepared on a rectangular ground of width *W* and height *H* in a golden ratio: roughly 50cm wide by 81cm tall. The catenary curves that form the quilted stitchings are suspended from the upper boundary, and have their vertices (lowest points) somewhere within the image boundaries. The images are generated entirely within the Wolfram *Mathematica*[iv] computational environment using a generative process guided by aesthetic and heuristic constraints.

Individual Catenary Curves

The analytical description of a catenary (hanging chain) is given by

$$y = a\left[e^{x/a} + e^{-x/a}\right] = \; a\;\cosh\left(\frac{x}{a}\right)$$

in Cartesian coordinates.[v] The positive real constant *a* is related to the linear mass density of the chain and the tension in the chain at its lowest point (vertex). The hyperbolic cosine's value at $x = 0$ is 1, so we can normalize our catenaries to place the vertex at the origin:

$$y = a\cosh\left(\frac{x}{a}\right) - a$$

A standard transformation

$$x \to x - h, y \to y - k$$

then allows a rigid translation of the catenary with its vertex at (*h, k*):

$$y - k = a\cosh\left(\frac{x-h}{a}\right) - a$$

While this provides a compact description of a catenary, the choice of *a* is nonintuitive. We would rather determine a particular catenary curve by choosing the suspension points along the upper boundary and the location of the vertex. These choices implicitly determine the value of *a*; however, since we have a transcendental equation, there is no symbolic solution for *a*, but a numerical root-finding algorithm can be employed to find an approximation to *a*. To create a curve family that elicits a 3D quality, we transform the Cartesian description of the catenary into a parametric description in 3D, and consider the initial catenary as the silhouette of a 3D *catenoid*, a surface of revolution. These are reminiscent of the Gaudí towers at La Sagrada Família and Casa Mila. These catenoids are then reprojected back into 2D for quilting. Figure 1shows a prototype rendering in Mathematica of a basic ruled catenoid.

```
In[42]:= For[x = 0; a = 1.1; prot1 = Graphics[], x < Pi, x += π/24,
          {p1 = ParametricPlot[{t Cos[x], a Cosh[t/a]}, {t, -3, 3}, Axes → False];
           prot1 = Show[prot1, p1]}
         ]

In[43]:= Show[prot1]
```

Out[43]=

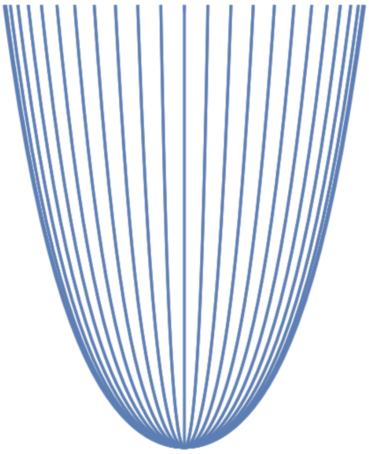

Figure 1. Prototype ruled catenoid surface projected to 2D.

Generative Composition: parametric description of 3D catenoid curve families

A particular instance of this generative process first requires the following choice:

- Number of catenoid curve families in composition (N)

Then, for each curve family, we must choose for the generating catenary:

- Suspension points (x_1, H) and (x_2, H),
- Vertex position (x_0, y_0), and
- Angular displacement $\Delta\omega$ of rulings about axis in surface of revolution.

Figure 2 shows a prototype rendering from Mathematica of two such catenoid curve families.

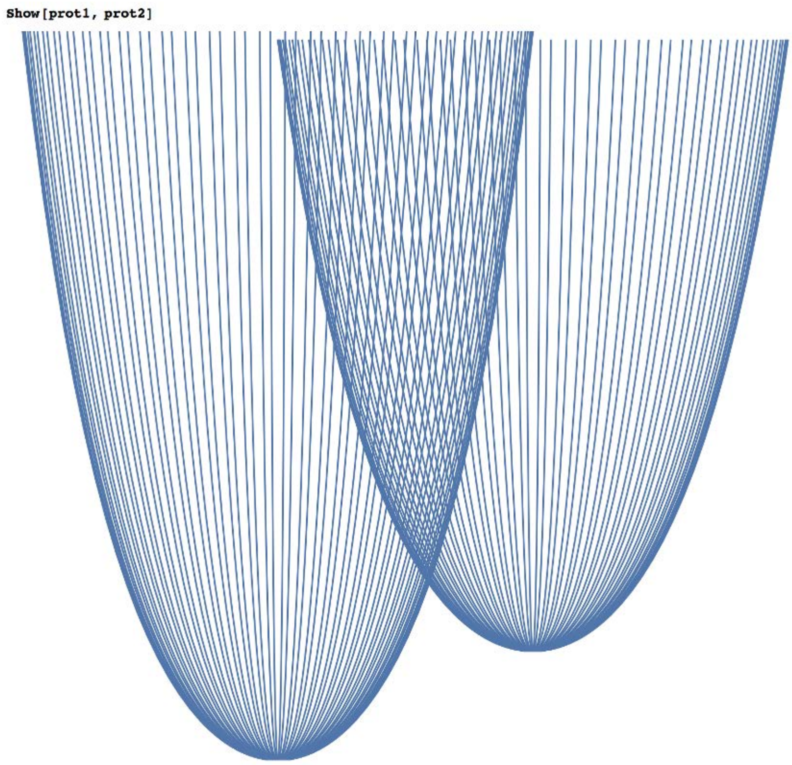

Figure 2. Two overlapping rendered catenoid curve families.

Aesthetic/Heuristic constraints on parameters

Based upon hard constraints and experimentation, we set

- $3 \leq N \leq 9$,
- $0 < x_0 < W$, $0 < y_0 < H$, etc.
- $0 < \Delta\omega < \pi$, generally chosen as π/n for $3 < n < 24$

The choice of N is governed by a need to create sufficient visual interest and adequate surface coverage, but without filling the image plane with so many stitches that the spatial ambiguity is overwhelming. Likewise, we add additional heuristics to provide a base image that covers a majority of the image plane, but which admits several other smaller catenoid families for visual interest. In our choice of $\Delta\omega$, we also need to provide sufficient visual cues to associate a family of curves following a given catenoid (rather than just a silhouette and one or two rules along the surface), but need to stop short of effectively "filling in" the entire catenoid.

Technique and Execution

Wolfram Research's *Mathematica* language is used to create, under our various constraints, the set of parameters that describe a given composition. The Cartesian descriptions are projected into 3D, and each catenary is used to produce a catenoid surface of revolution; we rule this surface at equally-spaced steps in 3D, and then project back onto a 2D surface.

Once one ruled form is drawn to dominate the composition but still be contained within the fabric, additional smaller catenoids are created. These are positioned relative to the initial form to create interesting overlaps and negative spaces, taking advantage of asymmetrical spacing criteria. *Mathematica* can export its graphic objects to SVG (Scalable Vector Graphic) format. For additional visual interest, we can export individual catenoid families as distinct SVG exchange files, thereby providing an opportunity to render these in different thread colors or materials. The corresponding SVG format exchange file is converted to an *Art and Stitch* interim file format (.ans) to make a digitized pattern for automated quilting. Here, the individual curves are connected to create one continuous line for uninterrupted sewing. The ANS file is converted to a .PAT file and imported into the ABM Innova quilting Machine's AutoPilot Mach 3 Software. The Pattern file is set to the parameters of the fabric height and width, then Mach 3 executes the sewing.

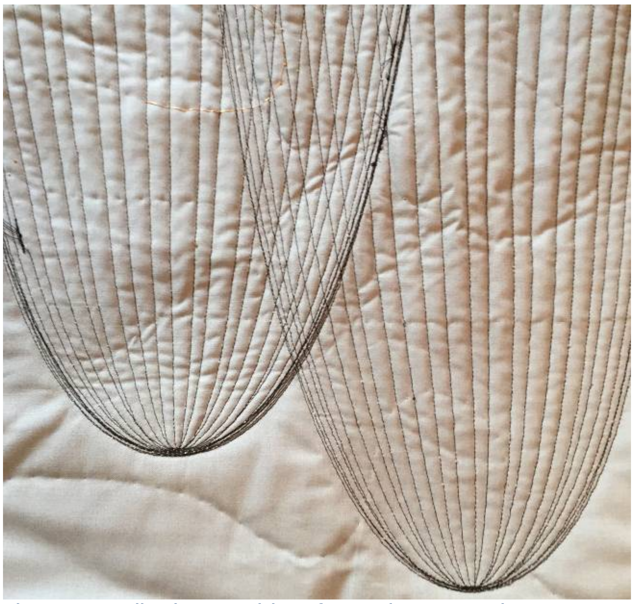

Figure 3. Quilted catenoid surfaces demonstrating trapunto effect

Our sample is sewn using Floriani metallic thread on top and Fil-Tec Glide Thread underneath on organza layered over a batik fabric and batting sandwich. The batik chosen for this work has a gradation from one hue to a more neutral tone adding further complexity to the visibility of the catenaries. In some places the stitching blends with the underlying fabric and sinks to the background while in others, the stitching contrasts and comes forward in space. This may or may not correspond with other depth cues such as *trapunto* effects and scaling. The background fill is hand-guided combining a progression of ruler work and pebbles.

Future Work

In the future, the fill patterns will be designed for automation to save time and to be more exact relative to the precision of the catenaries. Different thread weights and cording will be attempted to accentuate depth cues. Different scales at the minimum extremity will be tested to determine the limits of overlapping thread that can be used before breakage occurs. Hand-dyeing fabrics for more control of the underlying surface will also be a possibility.

Once these possibilities have been exhausted, we will add hemispherical protrusions to the catenary domes. This will be accomplished by adding 3-5 semicircular ruled lines along each ruled catenary line along the surface revolution. These will be projected onto 2.5 space. One of these will be created as a pattern file in *Art and Stitch*. The .PAT file can be repeated and each one stretched disproportionately.

Different fill patterns for automated use will be built using *Mathematica* to generate continuous line drawings to be realized in *Art and Stitch*.

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